

# TLA2B: A new validation tool for TLA<sup>+</sup>

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# Overview

- 1 Approach
- 2 Translation to B
- 3 Experiments
- 4 Demo
- 5 Conclusion

# Approach & Motivation

## Approach

Translating the non-temporal part of  $\text{TLA}^+$  to B

⇒ B does not support temporal formulas

## Motivation

- Validating  $\text{TLA}^+$  specification with existing B tools
- in particular: using the model checker PROB

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# TLA<sup>+</sup> & B-Method

|                   | TLA <sup>+</sup>       | B-Method                  |
|-------------------|------------------------|---------------------------|
| Invented by       | Leslie Lamport         | J.R. Abrial               |
| State-based       | ✓                      | ✓                         |
| Set theory        | ✓                      | ✓                         |
| Predicate logic   | ✓                      | ✓                         |
| Arithmetic        | ✓                      | ✓                         |
| Temporal formulas | ✓                      | X                         |
| State transition  | Before-after predicate | Generalised substitutions |
| Model checker     | TLC                    | PROB                      |
| Prove support     | TLAPS                  | AtelierB                  |

# MyHourClock

```
MODULE MyHourClock
EXTENDS Naturals
CONSTANTS start
VARIABLES hr
ASSUME start ∈ 1 .. 12

Inv  $\triangleq$  hr ∈ 1 .. 12
Init  $\triangleq$  hr = start
Inc  $\triangleq$  hr < 12 ∧ hr' = hr + 1
Reset  $\triangleq$  hr = 12 ∧ hr' = 1
Next  $\triangleq$  Inc ∨ Reset
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## Config file

```

INIT Init
NEXT Next
INVARIANT Inv

```



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```

Config file

INIT *Init*

NEXT *Next*

**INVARIANT** *Inv*

# Translation of the example

```
MODULE MyHourClock  
EXTENDS Naturals  
CONSTANTS start  
VARIABLES hr  
ASSUME  $start \in 1 .. 12$ 
```

```
 $Inv \triangleq hr \in 1 .. 12$   
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 $Inc \triangleq hr < 12 \wedge hr' = hr + 1$   
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```

```

MACHINE MyHourClock

```

```

END

```

# Translation of the example

MODULE *MyHourClock*

**EXTENDS** *Naturals*

CONSTANTS *start*

VARIABLES *hr*

ASSUME  $start \in 1 .. 12$

$Inv \triangleq hr \in 1 .. 12$

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MACHINE *MyHourClock*

END

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MACHINE MyHourClock
CONSTANTS start

```

END

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VARIABLES hr

```

END



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MACHINE MyHourClock
CONSTANTS start
VARIABLES hr
PROPERTIES start ∈ 1 .. 12

```

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PROPERTIES  $start \in 1 .. 12$ 
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INITIALISATION hr : ( $hr = start$ )

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OPERATIONS
  Inc_Op = ANY hr_n
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    THEN  $hr := hr\_n$  END

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# Concepts of typing

- TLA<sup>+</sup> is untyped, while B is strongly typed
- Type informations are needed for the translation
  - B requires type declarations of symbols
  - the translation of some operators depends on them

## Type inference algorithm (Hindley-Milner)

- Types of symbols can be inferred by considering the context the symbols are used
  - e.g.  $x$  must have the type Integer in the expression  $x + 1$
- some resulting restriction for the translation
  - variables must have a fixed type
  - only values of the same type are comparable

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# Supported TLA<sup>+</sup> values

- The following kinds of values exist in both languages:
  - integers, boolean values, strings, sets, functions, sequences, records
- Restrictions caused by the B type system
  - all elements of a set must have the same type
  - functions and sequences are based on sets
- Model values are translated using enumerated sets
  - ⇒ the translation conserves symmetry properties

# Supported TLA<sup>+</sup> operators

- Most TLA<sup>+</sup> operators can be mapped to B built-in operators
  - operators of the standard modules Naturals, Integers, Sequences, FiniteSets
- Other operators can be expressed by a combination of B operators
  - e.g. if-then-else
- User-defined operators are translated using B Definitions which are a kind of macro

# CHOOSE operator

Its general functionality can not be expressed in B

## Standard Module TLA2B

- contains useful operators such as minimum, maximum, sum and product of a set
- these operators can be mapped to B build-in operators

## Extending B

- PROB is able to load externally defined (polymorphic) functions
- semantics of the CHOOSE operator is expressed in this way

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# Translator: TLA2B

- Automatic translation
- The frontend is based on the parser SANY
- Reuse of TLC's configuration file parser
  - constant/operator assignment and replacement
- Stand-alone translator

# Integration of TLA2B into ProB

Applying PROB to TLA<sup>+</sup> specification

## PROB

- Model Checking
- Constraint-based checking
- Test generation
- Animation
- Visualization

## SimpleAllocator case study (by Merz)

- System to manage a set of resources (Rs) that are shared among a number of client processes (Cs)
- The specification allows TLC and PROB to use symmetry

| Cs | Rs | TLC         |          | TLA2B + PROB |          |
|----|----|-------------|----------|--------------|----------|
|    |    | no symmetry | symmetry | no symmetry  | symmetry |
| 3  | 2  | <1 s        | <1 s     | 2 s          | <1 s     |
| 4  | 3  | 28 s        | 2 s      | 678 s        | 8 s      |
| 5  | 3  | 450 s       | 29 s     | -            | 28 s     |
| 6  | 3  | >4200 s     | 573 s    | -            | 90 s     |

- Without symmetry TLC is superior to PROB
- For larger set sizes PROB's symmetry outperforms TLC

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# Constraint solving

## N-Queens:

- Searching for **all** valid placements of N queens on an  $N \times N$  chessboard so that no two queens attack each other
- The solution is encoded in a declarative way

| N  | Solutions | TLC    | TLA2B + ProB |
|----|-----------|--------|--------------|
| 6  | 4         | 1 s    | <1 s         |
| 7  | 40        | 16 s   | <1 s         |
| 8  | 92        | 375 s  | <1 s         |
| 9  | 352       | 2970 s | <1 s         |
| 11 | 2,680     | -      | <1 s         |
| 13 | 73,712    | -      | 41 s         |

- PROB is superior to TLC
- searching for a single solution:  
PROB finds a solution for  $N = 50$  in less than a second

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# TLA2B + PROB

## Demo

- PROB: <http://www.stups.uni-duesseldorf.de/ProB/>
- TLA2B: <http://nightly.cobra.cs.uni-duesseldorf.de/tla/>
- Logic Calculator: <http://cobra.cs.uni-duesseldorf.de/evalB/>

## Conclusion & Future work

- Translator TLA2B
  - type inference algorithm
  - supports a large subset of TLA<sup>+</sup>
- Integration of TLA2B into PROB
  - gain a new tool for TLA<sup>+</sup>
  - complementary to TLC
- all B tools can be apply to the translated B machine

### Future work:

- Testing the correctness of the translation
  - comparing the state spaces generated by TLC and PROB
- Improving and extending the translation
  - support for recursive function

# Questions?

# if-then-else

- If-then-else is an expression in TLA<sup>+</sup>  
e.g.  $\text{IF } x = 0 \text{ THEN } 1 \text{ ELSE } 1/x$
- Cannot use B's if-then-else substitution for translation

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- 1 in case of boolean type  
 $(P \Rightarrow e_1) \wedge (\neg(P) \Rightarrow e_2)$

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$$(P \Rightarrow e_1) \wedge (\neg(P) \Rightarrow e_2)$$

- 2 other

$$(\lambda t.(t \in \{\text{TRUE}\} \wedge P|e_1) \cup \lambda t.(t \in \{\text{TRUE}\} \wedge \neg P|e_2))(\text{TRUE})$$



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$$(\lambda t.(t \in \{\text{TRUE}\} \wedge P|e_1) \cup \lambda t.(t \in \{\text{TRUE}\} \wedge \neg P|e_2))(\text{TRUE})$$

# if-then-else

- If-then-else is an expression in TLA<sup>+</sup>  
e.g. IF  $x = 0$  THEN 1 ELSE  $1/x$
- Cannot use B's if-then-else substitution for translation

Translation of IF  $P$  THEN  $e_1$  ELSE  $e_2$

⇒  $e_1$  and  $e_2$  must have the same type

- 1 in case of boolean type

$$(P \Rightarrow e_1) \wedge (\neg(P) \Rightarrow e_2)$$

- 2 other

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# Records

- In B the Type of a record depends on
  - the fields of the record
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- How to translate  $[a \mapsto 1] = [b \mapsto TRUE]$ ?

$$rec(a : 1) = rec(b : TRUE)$$

$$rec(a : \dots, b : \dots) = rec(a : \dots, b : \dots)$$

- get a field  $x$  of a record  $r$  (TLA<sup>+</sup>:  $r.x$ )

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